ON THE DOMAIN SPECIFICITY OF THE HUMAN LANGUAGE FACULTY
AND THE EFFECTS OF PRINCIPLES OF COMPUTATIONAL EFFICIENCY:
CONTRASTING LANGUAGE AND MATHEMATICS

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ABSTRACT
The growth of language in the individual is determined by genetics, experience and principles of computational efficiency. The latter are taken to be part of natural laws affecting the development of biological systems. We discuss the effect of two principles of computational efficiency applying to the derivation of linguistic expressions and their interface representations. We develop the hypothesis that these principles are domain specific. They apply to language computations, but not to other cognitive computations. In this perspective, we contrast language and mathematics. We focus on indirect recursion, the recursive merger of a given projection X mediated by a functional element F: \([X \ F X]\). We posit that indirect recursion is forced by the principle of efficient computation Minimize Symmetrical Relations, whereas the intermediate functional head is not necessarily legible at the sensorimotor interface as enforced by the principle of Minimize Externalization. We discuss the results of psycholinguistic studies on the processing of complex nominals in English, which bring experimental support to our hypothesis. Furthermore, we provide evidence that indirect recursion, enforced by Minimize Symmetrical Relations and Minimize Externalization, holds for complex numerals, according to language specific parameters, differentiating, for example, Russian from Arabic. The facts indicate that indirect recursion is characteristic of the computational procedure of the human language faculty, while concatenation is available for mathematical operations in humans and animals. We discuss recent contributions of neuroscience with respect to the identification of brain pathways for language and mathematical computation. Theoretical and experimental results indicate that Minimize Symmetrical Relations and Minimize Externalization affect the computation and the processing of linguistic expressions by the human brain whereas there is no evidence that this would be the case for mathematical formulae.

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1. PURPOSE

Our research aims to understand grammar within a broader biolinguistic perspective, on the basis of notions that have been shown to shed light on the dynamics of complex systems such as biology and physics, namely the notions of symmetry, asymmetry and symmetry-breaking. These notions may also bring light to the domain specificity of human language as well as on language development. We will assume the general Minimalist framework according to which the operations of the language faculty are reduced to a minimum (Chomsky 1995 et seq).

Merge is the core operation of the language faculty. It is a binary and recursive operation. The sub-procedure of Merge includes an operation Select that selects items form the numeration (N), viz., a set of items retrieved once from the lexicon and transfers them into the workspace for further computation. The operation Agree is also part of the sub-procedure of Merge. It applies in feature valuing, and generally leads to displacement. The items from the numeration are associated with formal, semantic and phonetic features. Formal feature can be valued or unvalued. Unvalued features must be valued in the syntactic derivation before the expressions they are part reach the interfaces and thus satisfy the Principle of Full Interpretation. According to this principle, only expressions including valued features can be interpreted by the external systems, semantic and sensorimotor.

According to Chomsky (2005), the growth of language in the individual is determined by genetics, experience and principles of computational efficiency. The latter are taken to be part of natural laws affecting the development of biological systems. We focus on the role of the principles of efficient computation, which we claim to be domain specific to the language faculty. That is, these principles apply to language computation and not to other computations of the human cognition. More specifically, we propose that the principles of Minimize Symmetrical Relations and Minimize Externalization are part of the principles of efficient computation along with other principles, such as Derivation by phases, which limits the computational load of syntactic computation, and Minimal Link, which limits the search space of the syntactic operations (Chomsky 1995, 2001, 2005, 2013, a.o.).

Minimize Symmetrical Relations and Minimize Externalization subsume other principles that have been proposed as well as they cover additional cases.

Minimize Symmetrical Relations applies as early as possible in the syntactic derivations and eliminate symmetrical relations. In syntactic derivations, symmetrical (sisterhood) relations can be derived by Merge. Minimize Symmetrical Relations will eliminate the symmetry by displacing one or the other constituent (see Moro 2000 et seq.). Furthermore, Minimize Symmetrical Relations applies to the operation Select. This operation applies to the elements of a numeration and transfers them into the workspace for further computation Minimize Symmetrical Relations ensures that the bundles of features of the selected elements be in a proper inclusion relation (see Di Sciullo 2005a, Di Sciullo and Isac 2005). Minimize Symmetrical Relations contributes to the computational efficiency of Merge, eliminating non-converging derivations that could be derived if the computational procedure of the language faculty would be free. Thus, third factor principles apply to derivations and ensure computational efficiency. Third factor principles also apply in language development and may provide deeper explanation to phylogeny and language impairments, as discussed in Di Sciullo and Aguero.
(2008) and Di Sciullo (2011). We show that this principle applies in the derivations of linguistic expressions, including recursive nominal compounds and complex numerals in sections 2 and 3 below.

Minimize Externalization limits the pronunciation of linguistic elements if their features can be recovered from their local syntactic context. Minimize Externalization covers the principle Chomsky (2013) refers to as Pronounce the Minimum, to derive the fact that copies of displaced constituents are generally unpronounced. Minimize Externalization applies to functional heads, as it is the case for functional projections where adjuncts, such as adverbs and adjectives, occupy the Specifier position of a functional heads. Minimize externalization also covers silent constituents, including the silent counterparts of the nouns years and hour (and age and time) in the syntax of determiners, as discussed in Kayne (2005) in expression such as it is six \( \langle \text{…CLOCK TIME}_{i} \text{ F}^{o} \text{ [six HOUR] }_{t} \rangle \) and it is tree. It also covers null subjects and objects in Pro-drop languages (Rizzi 1982, 1986, Platzaek 1987, Jaeggli and Safir 1989, Rizzi and Shlonsky 2007, Camacho 2010, Roberts and Holmberg 2010). We argue that this principle plays a role in the non-pronunciation of functional heads in recursive nominal compounds and complex numerals in section 2 and 3 below.

We thus focus on the effects of Minimize Symmetrical Relations and Minimize Externalization in the derivation of recursive structures. We contrast recursion in language and mathematics, and suggest that Principles of efficient computation, such as the principles above, apply to language, whose expressions are computed by specific areas of the human brain, BA44 and BA45, and not to mathematics, which expressions are computed by different areas of the brain. We provide support to the hypothesis in (1), whose qualifies further Chomsky, Hauser and Fitch’s (2002) position according to which unbounded recursion is unique to the human Language Faculty.

\[(1) \text{ Indirect recursion is a distinctive property of the computational procedure of the Language Faculty.}\]

The recursive merger of a given projection X mediated by a functional element F generates indirect recursion: \([X \ F X]\). We posit that indirect recursion is forced by the principle of efficient computation Minimize Symmetrical Relations, whereas, the intermediate F head it is not necessarily legible at the sensorimotor interface as enforced by the principle of Minimize Externalization. We develop the hypothesis that these principles are domain specific. They apply to language computations, but not to other cognitive computations. In this perspective, we contrast language and mathematics.

This paper is organized as follows. Firstly, we discuss the properties of Merge and indirect recursion, and argue that principles reducing complexity apply to the sub-operation of Merge: Select. We bring empirical evidence for (1) on the basis of recursive nominal compounds in Brazilian Portuguese. Secondly, we contrast the properties of recursion found in language, with the properties of recursion found in mathematics. Finally, we discuss brain-imaging studies pointing to the domain specificity of language as opposed to general reasoning, including mathematics.
2. MERGE AND INDIRECT RECURSION

Merge (Chomsky 1995, 2001, 2013) is the core operation of the Language Faculty. It is a binary and recursive operation (Roep 2011, Arsenijevic & Hinzen 2011, Di Sciullo 2014, Maia et al., to appear, a.o.). While recursion is generally defined as being the property of a rule to reapply to its own output, it may also refer to the replication of a given category X, such as in sentential embedding. Indirect recursion unifies these two notions of recursion.\(^2\) We define it as follows:

\[ \text{(2) Indirect recursion is the recursive merger of } X \text{ mediated by a functional category } F. \]

Merge is defined as in (3) in Chomsky (1995). Chomsky (2001) distinguishes between External and Internal Merge. External Merge applies to two separate syntactic objects, whereas if either of them is part of the other it is Internal Merge. Indirect recursion gives rise to configurations such as the one in (4), where X and Y are categorically non distinct, in the course of the derivations.

\[ \text{(3) Merge (Chomsky 1995)} \]

Target two syntactic objects α and β, form a new object \( \Gamma\{\alpha, \beta\} \), the label LB of \( \Gamma(\text{LB}(\Gamma)) = \text{LB}(\alpha) \) or \( \text{LB}(\beta) \).

\[ \begin{array}{cc}
(4) & \text{a.} & \text{F} & \text{b.} & \text{X} \\
& X & F & & X & X \\
& F & X & & \end{array} \]

In configurations such as (4a), there is an asymmetry between the replicated category; whereas this is not the case in (4b). The asymmetry is structural, as can be expressed in terms of the asymmetrical c-command relation.

\[ \begin{array}{cc}
(5) & \text{C-command: } X \text{ c-commands } Y \text{ iff } X \text{ and } Y \text{ are categories and } X \text{ excludes } Y, \text{ and every category that dominates } X \text{ dominates } Y. \\
& \text{Asymmetric c-command: } X \text{ asymmetrically c-commands } Y, \text{ if } X \text{ c-commands } Y \text{ and } Y \text{ does not c-command } X. \text{ (Kayne 1994)} \end{array} \]

Our contention is that indirect recursion is enforced by the principle of efficient computation \textit{Minimize Symmetrical Relations.}

In the Dynamic Antisymmetry framework (Moro 2000 and seq.) points of symmetry may arise in the derivation of linguistic expressions and displacement must apply to break the symmetry. Canonical and inverse copular constructions constitute the core case discussed by Moro. Thus, (5a) and (5b) are derived from (6) by the displacement of the constituents in the domain of the auxiliary to the subject position (Specifier of TP).

\[ \text{2. Center embedded and tail recursions are structurally distinct. They are observed in phrasal syntax as well as under the world level. See Di Sciullo (2014) for discussion.} \]
(6) a. The picture on the wall was the cause of the riot.
b. The cause of the riot was the picture on the wall.

(7) a. [ was [[the picture on the wall][the cause of the riot]]
b. [[the picture on the wall][ was [[the picture on the wall][the cause of the riot]]]c. [[the cause of the riot]] was [[the picture on the wall][the cause of the riot]]

In this framework, indirect recursion is derived by the operation Internal Merge and not by External Merge. The two DPs in (6a) are External Merge, the auxiliary is Externally Merged to the previously derived structure, then one of the two DPs is displaced by Internal Merge to the Specifier of TP. In the resulting structure, the higher DP asymmetrically c-commands the lower DP and a functional category, here an auxiliary, intervenes between the two non-distinct categorical structures, the two DPs.

In other framework, for example in the Antisymmetry framework (Kayne 1994 and seq.), no point of symmetry can be generated in syntactic derivations. The merger of two maximal projection, whether they are categorically distinct or not, is never direct. A maximal projection must be first merged to a functional category before the complex merges with another maximal projection. This can be represented by using ordered pairs, represented in (9) with angle brackets, assuming, as in Kayne (2011), that Merge derives precedence relations in addition to dominance relations.

(8) a. *<XP, YP>
b. <F, YP>
c. <XP, <F, YP>>

In the Asymmetry framework (Di Sciullo 2005a, et seq.) asymmetrical relations are core relations of the language faculty. They play a predominant role in the derivation of linguistic expressions and at the interfaces with the external systems. Assuming, as in Moro (2000) that points of symmetry (sisterhood relations) may arise in the derivations and either one or the other constituent in a sisterhood relation can be displaced without altering acceptability (*) or semantic interpretation, then points of symmetry are not observed in the derivation of compounds. The parts of compounds cannot be commuted without a difference in interpretation (#). Similar restrictions apply on recursive compounds, as illustrated in (11), and discussed in Di Sciullo (2011).

(9) a. [ X Y ]
b. *# [ Y X ]

(10) a. a blood test
b. *# a test blood
c. a clinic test
d. *# a test clinic
The expressions in (10) qualify as compounds, as defined in Di Sciullo and Williams (1987). The categorical and semantic head of the compound is on the right. They bear one primary stress, which falls on the non-head. Their semantics is partly compositional. In Di Sciullo (2005b), we proposed that the semantic relation between the constituents of compounds is covert and can be expressed in terms of a functional category, such as OF and FOR, relating the constituents. This functional category would carry formal and semantic features, which contribute to syntax and the semantics of these expressions. This follows from the indirect recursion hypothesis.

Thus, structures such as the one in (12), where an intermediate functional head intervene between two nominal categories, is an instance of indirect recursion generated by the computational procedure of the language faculty. Moreover, such structures satisfy the Interface Transparency thesis according to which there is a one to one mapping between syntactic representations and semantic representations (Uriagereka 2008, Lohndal 2014, a.o.).

The fact that the intermediate functional projection in NN compound must be pronounced in some language brings empirical support to the indirect recursion hypothesis. This is the case for NN compounds in Brazilian Portuguese. The examples in (13)-(16) show that, contrary to languages such as Italian, where a preposition may not overtly relate the nominal constituents of nominal compounds, this is not the case for Brazilian Portuguese, as well as European Spanish (controle de pasaportes), where a proposition must be pronounced.

(11) a. an clinic drug test  
    b. *# a drug clinic test  
    c. an hospital expert tested drug  
    d. *# an expert hospital tested drug

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The obligatory presence of a preposition in Brazilian Portuguese nominal compounds is the consequence of the basic asymmetry of Merge, whereby two maximal constituents cannot be merged together directly. Each constituent must first be merged with a functional head F. This is enforced by the principle of efficient computation \textit{Minimize Symmetrical Relations}.

\subsection*{2.1 Merge, indirect recursion and third factor principles}

\textit{Minimize Symmetrical Relations} provides further explanation as for why the sub-procedure of Merge restricts the application of this operation. If Merge was not so restricted, several derivations would be possible, most of them yielding not legible interface representations. For example, Di Sciullo and Isac (2008) show that Free Merge may build many unattested structures from the same set of terminals, some of which do not capture basic syntactic relations. Free Merge may fail to capture the predication relation between the subject and the verb, as the subject may be merged in a lower position than the predicate, it may fail to derive full arguments, as determiner may not be merged with their nominal complement, as well as it may fail to capture the relation between arguments and adjuncts, as adjunct could be merged lower than arguments. Contrasting with Free Merge, Asymmetric Merge is an operation that applies to a pair of elements in the Numeration whose sets of features are in a proper inclusion relation (Di Sciullo and Isac 2008). The criterion for deciding the order in which items in the Numeration must be Merged is the proper inclusion relation: the set of features of the merged item must stand in a proper inclusion relation with the set of features of the object derived in the workspace. The order of application of Merge follows from the restriction on Select. Furthermore,
Asymmetric Merge derives indirect recursion for free. Given Asymmetric Merge, Select cannot merge two constituents with identical matching features. The smallest proper sub-set requirement ensures that the merger is asymmetrical. Thus, the proper inclusion restriction on Select, Merge will derive the configuration in (18) on the basis of the numeration in (17) for a recursive nominal compound in Italian, such as *punto controllo passaporti*, where the intermediate functional projections are not pronounced, enforced by the principle *Minimize Externalisation*.

\[(17) \quad N = \{ \text{punto: [N] [uF]; controllo: [N] [uF]; passaporti [N]; F [F], [uN] } \}
\]

\[(18) \]

The derivation of nominal compounds in Romance languages is enforced by the principles of *Minimize symmetrical relations*. One consequence of this principle is that two nominal elements, including bare nouns, are first merged with a functional head before the resulting structure is merged with another nominal category. This ensures the derivation of hierarchical structure instead of flat structure for compounds. The effect of the principle of manifests itself in languages such as Italian, where the intermediate functional head may not be pronounced, as we illustrated above.

*Minimize symmetrical relations* and *Minimize Externalization* also apply in the derivation of compounds in languages such as English, where the non-head precedes the categorical and semantic head of the structure. According to the Asymmetry framework, the derivation of English nominal compound is the following. In the case of object-verb compounds, such as *passport control*, the object is externally merged with a functional F head, enforced by *Minimize symmetrical relations*. The unvalued nominal feature, [uN], of the lower F head is valued by the valued nominal feature, [N], of the object *passport*. *Control* is externally merged to the resulting structure, and its unvalued feature, [uF], is valued by the lower F head. A second functional F head is Externally merged to the resulting structure and the object is displaced by Internal Merge to the Specifier of the F projection and feature valuing eliminates un unvalued nominal feature of the higher functional category. The result of the derivation is represented in (19), where *control* is analyzes as a deverbal noun (N_{v}) and the set of features of the lower F head includes the semantic feature OF. This feature is not pronounced in the derivation of English compounds, enforced by the Principle of *Minimize Externalization*. 
Thus, one consequence of the Indirect Recursion hypothesis is that function heads are part of the argument structure as well as they are part of the higher layers of syntactic projections, as enforced by efficiency principle of *Minimize Symmetrical Relations*; whereas these functional heads may not be pronounced in some case, enforced by *Minimize Externalization*.

### 2.2 Psycholinguistic experiments

Third factor principles may also provide further explanation for differences in language processing. We highlight some aspects of a psycholinguistic experiment on the acceptability of internal argument-verb vs. modifier-verb compounds in English, as illustrated in (20a) vs. (20b), reported in Di Sciullo and Tamioka (2011). 4

(20) a. the meat-cutting knife
   b. the finger-painted portrait

The noun-verb compound is a saturated predicate and hence it appears as a participle with *-ing*, combining with a noun that is interpreted as the subject (20a). The noun-mod-verb compound is an unsaturated predicate and hence it appears as a passive participle with *-ed*, combining with a noun that is interpreted as the object (20b). The morphology of the deverbal constituent in (20a) is homophonous.

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3. Maia et al. (forthcoming) discuss the results of oral sentence/picture matching experiments and ERP experiments comparing the processing of recursive PP embedding and coordination in Karaja and in Portuguese. The objective of the experiments was to assess whether the processing of PP recursion is costlier than the processing of PP coordination. Recursion is the result of a syntactic algorithm that is costly to be launched, but once it is established, it undergoes habituation and does not pose any extra significant effort to the system. The facilitation could be interpreted as a performance or third factor phenomenon: once engaged in the syntactic recursive algorithm, subsequent embeddings would be facilitated. See also Trotzke, Bader & Frazier (2013) for recursion and third factors effects.

4. The asymmetry between objects and adjuncts has received much attention in works on compound formation (e.g., Baker 1988, Rosen 1989, Rivero 1992, Kuiper 1999, Spencer 2011, Di Sciullo 2005b, 2011, a.o.). A major puzzle concerning compounds in language such as English is that even though Head-movement captures the compound formation of object-verb type (Baker 1988), it cannot account for the presence of adjunct-verb compounds. This is not the case in the Asymmetry based analysis of compounds discussed in this paper as well as in Di Sciullo (2011, 2014).
with the progressive aspect marker -ing, and the morphology of the deverbal constituent in (20b), with the simple past tense -ed.

In a modifier-verb compound, the adjunct is external to the verbal projection, whereas in the object-verb compound, the object (int. argument) is internal to the verbal projection, as illustrated below:

(21)

\[
\text{modifier} \quad \rightarrow \quad V \quad \rightarrow \quad \text{int. argument}
\]

The experiment probed the effect of a configurational argument/modifier asymmetry in novel compounds. In this experiment, we focused on the following difference between modifier-verb and argument-verb compounds: \( N_{\text{arg}} - V \) compounds take the participle morpheme –ing, in contrast, \( N_{\text{mod}} - V \) compounds take the –ed morpheme. In addition, we hypothesized that the effect of homophony is expected between the participle morphology (ed and ing) and the tense/aspect suffix (-ed and -ing) on the acceptability of novel compounds. Thus, the presence of \( N_{\text{arg}} - V \) compound, \( N_{\text{mod}} - V \) compound representation in the mental representation will form the basis for the effect of homophony.

The stimuli are mono-transitive verbs and a noun forming 20 novel N-V compounds (10 in each class). In order to control for the potential effect of frequency in acceptance rating, all the constituents (nouns and verbs) of the compounds are taken from the most frequent word lists. In addition, the compounds are presented in a sentence to help disambiguate the classification of the compound. In the experiment, each compound appears in three different contexts. The control context is the participle use, in which the compounding is most productive, and two verbal contexts with different tense/aspect morphology (-ing or –ed), and 60 fillers are used. A sample of the stimuli is presented below:

(22) Object-V
a. The dreamer star-counted all night.
b. The traveler bird-caught in the back yard.

Adjunct-V
c. The valet sand-parked the client’s car.
d. The pilot desert-flew the small plane.

Fillers
e. The girl turned on the clock-light on the wall.
f. The penguin met her pole-sister after the storm.

The following pattern emerged from this experiment: With \( N_{\text{arg}} - V \) compounds, the subjects rated the –ing forms significantly more acceptable than the –ed forms. With \( N_{\text{mod}} - V \) compounds, the subjects showed preference for the –ed form over the –ing form. What is interesting here is that the tendencies of these two types of novel compounds go in opposing direction, which indicates that there is an
effect of homophony. Crucially, the observed pattern cannot be due to the nature of the suffixes alone as there is no relation between the choice of tense/aspect morphemes and the semantics of the noun in the compound. The effect of homophony/increased acceptability differentiates $N_{\text{arg}}$–$V$ compounds from $N_{\text{mod}}$–$V$ compound categorically.

The means of acceptability rating for each type and context is as follows:

<table>
<thead>
<tr>
<th></th>
<th>$N_{\text{arg}}$–$V$</th>
<th>$N_{\text{mod}}$–$V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>past tense (-ed)</td>
<td>3.43</td>
<td>2.74</td>
</tr>
<tr>
<td>progressive(-ing)</td>
<td>2.72</td>
<td>3.45</td>
</tr>
</tbody>
</table>

The results of our experiment show a sharp difference in acceptability judgment. We take this as indicating that humans perceive and interpret compound on the basis of their internal $N_{\text{arg}}$ vs. $N_{\text{mod}}$ configurational asymmetry. The sharp differences in acceptability between novel $N_{\text{obj}}$ vs. $N_{\text{modifier}}$ compounds cannot be attributed to frequency, since the compounds are novel. Moreover, the parts of the compounds are drawn from most frequent word lists. Differences in acceptability of novel compounds suggest that human processing is sensitive to configurational asymmetries, including the object–adjunct/internal argument–modifier asymmetry. This asymmetry influences how compounds are processed.

The result of the experiment provides evidence of a configurational asymmetry between objects and adjuncts in compounds. This study provides additional experimental evidence to the findings of Tsapkini, Jarema, and Di Sciullo (2004) where priming showed a significant difference in reaction time between internal vs. external aspectual prefixed verbs in French. Together these results suggest that human processing of complex words accesses asymmetric relations, such as the argument/modifier, and the internal/external aspect asymmetries. Interface representations provide the asymmetrical bases for legibility, which include configurational asymmetry, and are not limited to linear precedence relations or frequency effects.

The principles of efficient computation, Minimize Symmetrical Relations and Minimize Externalization provide a mean to go beyond explanatory adequacy. Minimize Symmetrical Relations provides a
justification as for why asymmetrical relations are characteristic of compounds notwithstanding the compounds may include apparently adjacent constituents which are non distinct categorically. *Minimize Externalization* provides a rationale as for why copies of displaced constituents as well as for why intermediate functional heads are not pronounced in object-verb and adjunct-verb compounds in languages such as English as well as in other languages. The results of psycholinguistic experiments suggest that these principles may also intervene in the processing of linguistic expressions.

2.3 Section summary

Recursion is a property of Merge to reapply to its own output. Recursion can also be thought of as replication of the same categorial structure. NN compounds in BP provide evidence that indirect recursion derived by Merge and is enforced by the principles of efficient computation *Minimize Symmetrical Relations*. The non-pronunciation of F is enforced in BP is enforced by *Minimize Externalization*. Experimental results for acceptability tests suggest that configurational asymmetries may influence the way in which linguistic expressions with different internal structure are processed. The experimental results provide behavioral as well as neurolinguistic support to the hypothesis that the language faculty is sensitive to configurational asymmetries brought about by the recursive application of Merge.

3. RECURSION IN LANGUAGE AND MATHEMATICS

In this section, we compare language with arithmetic and argue that recursion is indirect in the generation of complex numerals, while it is direct in addition and multiplication as well as in counting, something non-human primate can also do with small numbers.

Comparative studies of mathematical capabilities in nonhuman animals indicate that many animals can handle numbers up to 6-7 (perhaps directly, or perhaps via subsidizing), but they cannot deal with greater numbers. Several experimental studies indicate that animal have elementary arithmetic operations such as addition and subtraction. For example, Hauser, MacNeilage and Ware’s (1996) results suggest that rhesus monkeys detect additive and subtractive changes in the number of objects present in their visual field. Given the methodological and empirical similarities, it appears that nonhuman primates such as rhesus monkeys may also have access to arithmetical representations. Rugani et al. (2009, 2011) demonstrated arithmetic in newly hatched chickens. The scientists reared the chicks with five identical objects, and the newborns imprinted on these objects, considering them their parents. But when the scientists subtracted two or three of the original objects and left the remainders behind screens, the chicks went looking for the larger number of objects, sensing that Mom was more like a three and not a two. These findings implicate that animal have a relatively sophisticated representational system in the absence of language. However, while animals and human count nonverbally, however there is a discontinuity between non-verbal and verbal counting, as discussed for example in Gelman and Cordes (2001). The ability to develop complex numerals is human-specific. Thinking beyond experience is a by-product of a uniquely human, non-adaptive,
cognitive capacity. In addition to human’s ability to individuate precise numbers as opposed to approximation, unbounded recursion could as argued in Chomsky, Hauser and Fitch (2000) be a determinant divide between human and animal ability for complex numeral and language. According to our hypothesis indirect recursion is human specific.

We start by identifying recursive sets in mathematics and associated recursive operations derived them.

3.1 Recursively defined sets

The canonical example of a recursively-defined set is given by the natural numbers. The set of natural numbers is the smallest set satisfying the two properties in (23).

\[
\begin{align*}
\text{(23)} & \quad \text{a. } 0 \text{ is in } \mathbb{N} \\
& \quad \text{b. If } n \text{ is in } \mathbb{N}, \text{ then } n + 1 \text{ is in } \mathbb{N}
\end{align*}
\]

Many mathematical axioms are based upon recursive rules. For example, the formal definition of the natural numbers by the Peano axioms can be described as: 0 is a natural number, and each natural number has a successor, which is also a natural number. By this base case and recursive rule, one can generate the set of all natural numbers. The Fibonacci (Fib) sequence is a classic example of recursion, (24). Recursively defined mathematical objects include functions, sets, and fractals in particular.

\[
\begin{align*}
\text{(24)} & \quad \text{a. Fib}(0) \text{ is } 0 \text{ [base case]} \\
& \quad \text{b. Fib}(1) \text{ is } 1 \text{ [base case]} \\
& \quad \text{c. For all integers } n > 1: \text{ Fib}(n) \text{ is } (\text{Fib}(n - 1) + \text{Fib}(n - 2))
\end{align*}
\]

Counting is a recursive operation. Chomsky (2008) defines counting in terms of Merge as follows. Suppose that a language has the simplest possible lexicon: just one lexical item (LI), call it ‘one’. Application of Merge to \{one\} call it ‘two’. The application of Merge to \{one\}yields \{one, \{one\}\} call it ‘three’ and so on. Merge applied in this fashion yields the successor function. (Chomsky 2008).

\[
\begin{align*}
\text{(25) Counting and the successor function} \\
1 = \{\text{LI}\}, 2 = \{\text{LI}, \{\text{LI}\}\}, 3 = \{\text{LI}, \{\text{LI}, \{\text{LI}\}\}\}, \ldots
\end{align*}
\]

A question that could be asked is whether the Merge yielding the successor function is External or Internal. If every application of Merge requires access to the lexicon, then Merge is external. If we limit the numeration to one item, a single term numeration would permit only one selection. The computational system would require that the system recycle what is already found in the current state of the structural description. That would be Internal Merge (Bolender 2011).

Several works seems to indicate that External Merge is not human specific (Byne and Russon (1998) mountain Gorilla food preparation, McGonigle et al. (2003) capuchin monkeys ordering objects,
Seyfarth et al. (2005) Baboons knowledge of their companions, Schino et al. (2006) Japanese macaques knowledge of their companions, a.o.).

Counting would then be compatible with the hypothesis that Internal Merge, yielding quantification, Operator-variable structures, Opx (...x...), would be human-specific. Quantificational, Operator-variable structures, are necessary in theoretical languages, including terms that designate unobservable, contra observational languages. Uriagereka (2008) argues that, being a context-sensitive operation, Internal Merge places greater demand on working memory than does External Merge alone. The probe-goal search, required by Internal Merge, involves scanning the derivational record to find the object to be merged. External Merge does not require a scan of the derivational history. So less developed working memory in the other primitive species could suffice to explain their using Merge, but not Internal Merge (Coolidge & Wynn 2005). Accounting for the human uniqueness of Internal Merge could also explain the human uniqueness of open ended counting, assuming the restricted numeration hypothesis.

Chomsky’s (1988) discussion of counting as an abstraction of the Faculty of Language offers two reasons for which it should be the case, i) the development of the mathematical ability in different people, and ii) the improbability of a system exhibiting discrete infinity. Merge is crucial for counting as well as for thinking about the unobservable. The ability to develop complex numerals is human-specific. Thinking beyond experience is a by-product of a uniquely human, non-adaptive, cognitive capacity. The ability for the human mind to compute complex numerals is a consequence of the great leap from finite and continuous systems, such as the gesture system, to systems of discrete infinity, such as language, mathematics and music. Recursion is observed in language and mathematics. Indirect recursion is language specific.

3.2. Complex numerals

The ability for the human mind to compute complex numerals is a consequence of the great leap from finite and continuous systems, such as the gesture system, to systems of discrete infinity, such as language, mathematics and music. In this section, we argue that while recursion is observed in language and mathematics. Indirect recursion is language specific. We raise the following questions: In what sense is indirect recursion domain specific? How different are language and mathematics computations? How are these computations implemented in the brain?

Complex numerals are not strings of numerals. They cannot be derived by operations on strings, such as the concatenation operations, since concatenation does not keep track of the properties of the concatenated elements. Complex numerals are hierarchical constituent structure, signaled by the presence of a functional category or an intonational pause boundary in complex numerals, as in (26a) with the coordinating conjunction relating the two number phrases, and in (26b) with an intonational pause between the phrasal constituents. Hierarchical constituent structure is signaled by intonation pauses.

5. In the theory of computation, the concatenation operation on strings is generalized to an operation on sets of strings as follows: For two sets of strings $S_1$ and $S_2$, the concatenation $S_1S_2$ consists of all strings of the form vw where v is a string from $S_1$ and w is a string from $S_2$. 
Intonational boundaries tend to occur before syntactic constituents (Lehiste, 1973; Gee & Grosjean, 1983; Ferreira, 1991). They are also constrained by the Sense Unit Condition (Selkirk, 1984). A sense unit is defined as a constituent formed by a head and, optionally, the head of any number of its modifiers and/or arguments. The Sense Unit Condition of Intonational Phrasing: The immediate constituents of an intonational phrase must together form a sense unit. Two constituents Ci, Cj form a sense unit if Ci depends on Cj. Watson and Gipson (2005) report psycholinguistic experiments indicating that speakers tend to place boundaries before and after large syntactic constituents. Listeners use intonational boundaries as cues that signal where to make attachments.

Several analyses have been proposed to derive the hierarchical structure underlying complex numerals.6 We will consider cardinals. According to Ionin & Matushansky (2006), cardinals, both simplex, like four, and complex, like four hundred in expressions such as four books and four hundred books, are semantically modifiers. They are of the semantic type <<e,t>, <e,t>>, and take an NP argument.

According to Stavrou & Terzi (2008) in Greek, Simplex and complex cardinals are generated in theSpecifier of NUMP. This accounts for their recursive structure, contra Qs. With cardinals containing Numerical Nouns, recursion is located in the complement position of NUMCL, as the tree in (28), from Stavrou and Terzi (2008) illustrates.

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Ionin and Matushansky (2006) provide different syntactic analyses for complex cardinals. Whether they involve multiplication or addition. Complex cardinals involving multiplication, as it is the case for expressions like *two hundred* are analyzed as complementation, whereas complex cardinals involving addition, like *one hundred and two*, are two simplex cardinals combined into one via coordination. A unified analysis would capture the fact that they are both complex cardinals, differing minimally in their feature structure.

Stavrou and Terzi (2008) analysis fails to express the difference between complex cardinals involving addition and complex numerals involving multiplication. This can be seen in the structure in (28), where NUM, the Head of NUMP, is associated with the phi-feature plural [± PL] and no other feature specific to complex numerals involving multiplication. Furthermore, while Stavrou and Terzi (2008) locate the recursion in complex cardinals in the complement position of NUMP, no principled reason is provided as for why it should be located in that position.

A unified analysis of complex numeral can be formulated on the basis of indirect recursion, to which we now turn.

### 3.3 Complex numerals and indirect recursion

Indirect recursion is evidenced by intervening functional projections. Numerals combine via an overt functional category in numerals involving addition as well as multiplication in some languages.
The pronunciation of the coordination conjunction is subject to cross-linguistic variation. In Russian, numerical expressions never contain an overt conjunction, while in other languages, as it is the case for Arabic, an overt conjunction is obligatory for addition (Zabbal 2005).\(^7\) Yet, in other language (e.g. English, French, Italian) the conjunction is pronounced in some cases. Empirical evidence that multiplicative structure also includes a functional projection comes from complex multiplicative structures in Romanian, (30), were the preposition DE (of), which is used independently in pseudo-partitive structures, (31), must be part of the recursive multiplicative structures (Brasoveanu 2007). Similar structures are found in English, as well as in the Romance languages, as the example in (32) illustrates.

(30)   doua sute de mii de carti (Ro)
      two hundred-PL DE thousand-PL DE books
      ‘two hundred thousands books’

(31) a.   doua sute de carti (Ro)
      two hundred DE books
      ‘two hundred books’

b.    o mie de carti
      one thousand DE books
      ‘one thousand books’

(32) a.    delle diecine di milliai di student (It)
      tens of thousands of students

b.    des dizaines de milliers d’étudiants (Fr)

c.    tens of thousands of students

These facts indicate that the structure of additive complex numeral is a conjunction structure. The conjunction is asymmetrical however. Their parts cannot be inverted without giving rise to gibberish, as in twenty one and *one twenty or a difference in interpretation, compare two hundred and hundred and two, in which case the derivations are distinct.

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\(^7\) The examples below from Zabbal (2005) show that the over conjunctions wa (and) is part of complex cardinals in Arabic in addition of Case markers. We will not discuss Case within complex numerals here.

i.   arba –at-u aalaaf –in wa- xams-u mi at-in rajul-in rajul-in
    four-NOM thousands-GEN and five-NOM hundred-GEN man-GEN
    ‘4500 men’

ii.  arba –at-u aalaaf –in wa- xams-u mi at-in rajul-in wa sitt-at-u rijaal -in
    four-NOM thousands-GEN and five-NOM hundred-GEN and six-FS-NOM men-GEN
    ‘4506 men’
Coordinations are asymmetric structures under an X-bar analysis (Kayne 1994, Munn 1987, Johannessen 1998, a.o.) or under an adjunction analysis (Munn 1993). According to the adjunction approach, XP is a projection of the first conjunct XP and XP dominates XP. The structure of coordination is asymmetrical.

(33) \[ \text{XP} \rightarrow \text{YP} \]
(34) \[ \text{XP} \rightarrow \text{ ConjJP} \]
(35) \[ \text{ConjJP} \rightarrow \text{XP} \rightarrow \text{ConjJP} \]

The occurrence of conjunctions in additive complex numerals indicates that they are asymmetrical hierarchical structures. The occurrence of a preposition in complex numerals involving multiplication also suggests that their structure is asymmetrical. The intervening preposition relates the two numeral phrases.

4.4 A minimalist analysis of complex numerals

Given Minimalist assumptions, feature valuation applies in the derivation of conjunctions as it does in the derivation of syntactic structures more generally. In addition to the cagetorial feature [Conj], their set of features includes two unvalued features, [uX]. These features are categorically identical, since conjunction cannot conjoin constituents of different syntactic categories. In addition, we will assume that the feature [AND] is part of their semantic features. Thus, conjunctions are associated with the features in (35). Thus, given the numeration in (36), the structure in (37), where feature valuation has applies, is derived. In the first step of the derivation, Conj is externally merged with XP, and the resulting structure is externally merged with YP in the second step of the derivation.

(35) Conj : [[Conj], [uX], [uX], [AND]]

(36) N: { XP [X], Conj [[Conj], [uX], [uX], [AND]], YP [X] }

(37) \[ \text{XP} \rightarrow \text{ConjJP} \rightarrow \text{YP} \]

While conjunctions are asymmetrical projections, some conjunctions are commutative, contrary to others. We propose that these two kinds of conjunctions have different derivational histories, which leads to the structure in (37) for the non-commutative cases and the structure in (38) for the commutative cases. In (38) the two conjuncts are externally merged in the first step of the derivation. In the second step of the derivation, the conjunction is externally merged with the resulting structure, and in the third step of the derivation one or the other conjunct is internally merged to the preceding structure.

(38) 
```
  XP
 /  \
XP   ConjP
   \  
    
      
Conj       XP
       \  
        [uX]  XP  YP
          \   
           [uX]  [X]  [X]
            \[AND]
```

In Di Sciullo (2012) we proposed that the structure of complex numerals is as in (39). In (39), the set of features of the functional category relating the two number phrases (NumP) includes two unvalued Num features [uNum]. This functional head is associated with the semantic feature [MULT] for multiplicative complex numeral, such as two hundred and the semantic feature [ADD] for additive complex numerals such as two hundred and two. The result of the derivations of additive and multiplicative structures, including feature valuing is represented below:

(39) a. NumP
```
  twenty

  [Num]

  F

  [Num]

  [ADD]
```

b. NumP
```
  two

  [Num]

  F

  hundred

  [Num]

  [MULT]
```

The compositional interpretation of the Conjunction F head is complex numerals is restricted to the interpretation of two operators features, addition (ADD) and multiplication (MULT). Complex numerals include unpronounced heads with ADD and MULT features. These features are legible by the semantic interface by the Conceptual-Intentional system and enable the conceptual interpretation of numerals.

The semantic features that are part of complex numerals are more restricted than the addition and multiplication operator of mathematics. One restriction is that the operator features ADD and MULT cannot relate two constituents of the same kind. For example, they cannot relate two lexical numbers, or two bases. This is evidenced by the expressions in (40), which are not complex numerals. In contrast, the mathematical operations of addition and multiplication are not so restricted. Furthermore, addition
and multiplication are commutative, whereas this is not the case for the parts of complex numerals, as (41) illustrates. Moreover, and central to our purpose, the recursive application of addition and multiplication operation in mathematics is not mediated by an intermediate element, as it is the case for the intermediate head in the recursive application of Merge, see (42). While the functional head, the conjunction and is pronounced in (42a), an intermediate symbol, for example the symbol \(^\) does not intervene between the parts of related numerical expressions, (42b). This suggests that indirect recursion is not a property of addition and multiplication, whereas it is a property of the operation of the language faculty, enforced by Minimize Symmetrical Relations. Such a principle in not part of mathematical computation and neither is Pronounce the Minimum, as mathematical operations do not generate covert elements.

(40) a. # one two, #ten twenty
    b. twenty one, one hundred
(41) a. twenty one, #one twenty
    b. 1+2, 2+1; 10x20, 20x10
(42) a. one hundred and one
    b. 101 vs. \(^1\)

Furthermore, the parts of complex numerals are not commutable without preserving the semantics of the complex numeral. This is illustrated in (43), where each numeral denotes a different natural number.

(43) a. two hundred
    b. hundred and two
    c. one hundred and two
    d. two hundred and one

This fact suggests that a derivation in terms of the displacement of one or the other constituent of the complex numeral in a higher position would be problematic. For example, while in (44a) the displacement of the first phrasal constituent derives an additive structure; this is not the case in (44b), with the displacement of the second constituent. Likewise in (44c), the displacement of the first phrasal constituent derives a multiplicative structure; however, in (44d), the displacement of the second constituent is not meaning preserving, hundred (and) two is an additive structure and not a multiplicative structure.
In the Asymmetry framework, complex numerals are derived by indirect recursion, as it is the case in as well as in extended recursive numerals, as in *two hundred (and) twenty two* and *twenty two hundred*. Unbounded recursive numerals are derived by the merger of two previously derived complex numerals, which cannot combine directly, but must be combined via a functional projection baring unvalued features to values, as well as semantic features, either [ADD] or [MULT]. The structures in (45) and (46) illustrate the result of the derivations. In (45a) External Merge combines the lexical number *two* with the functional F head whose unvalued feature have been valued. In (46a), with recursive complex numeral, an already derived complex numeral has been internally merged to the F head. Unbounded recursion is derived by the further external merger of an F head the preceding structure followed by a further external merger of a complex numeral, as in *two hundred twenty two* (222), *one thousand two hundred twenty two* (1,222), *one hundred one thousand two hundred twenty two* (101,222), *six millions one hundred one thousand two hundred twenty two* (6,000,101,222), and so on.

(45) a. NumP
   twenty
   [Num]
   FP
   F
   [Num]
   two
   [Num]
   [ADD]

   b. NumP
   two
   [Num]
   FP
   F
   [Num]
   hundred
   [Num]
   [MULT]

(46) a. NumP
   two hundred
   [Num]
   FP
   F
   [Num]
   twenty two
   [Num]
   [ADD]

   b. NumP
   twenty two
   [Num]
   FP
   F
   [Num]
   hundred
   [Num]
   [MULT]
Thus, according to this analysis, Numerals (NUM) merge with functional projections with valued features (ADD, MULT) and unvalued features (uNUM). Unvalued features are eliminated under the proper inclusion relation. Semantic features [ADD] and [MULT], be they pronounced or not, are legible at the interface with the Conceptual Intentional system.

The restrictions on the form of complex numerals are a consequence of Principles reducing complexity, namely Minimize Symmetrical Relations, enforcing indirect recursion, and Minimize Externalization, enforcing the non-pronunciation of the intermediate functional head in some language.

**Section summary**

Complex numerals are derived by the computational procedure of the language faculty. The merger of two numerals is mediated by a first merge with a functional projection, be it a coordination conjunction or a preposition. The core aspect of their semantics comes out form natural language semantics compositionality.

They exhibit the specific property of the recursion in human language: indirect recursion. This is not the case for arithmetic expressions. The operations of Mathematics are not subject to Minimize Symmetrical Relations and Minimize Externalization, which is the case for the operations of the Language Faculty.

The fact that complex numerals are composed of asymmetrical substructures headed by F heads suggests that they are also processed by the part of the brain that sub-serves language. The fact that these F heads correspond to arithmetic operators, even if not pronounced, indicates that the algebraic computation of complex numerals might also be provided by the part of the brain that sub-serves mathematical reasoning.

**4. DOMAIN SPECIFICITY OF LANGUAGE**

The Language Faculty is human specific. The core operation of this faculty derives hierarchical recursive structures. The processing of hierarchically complex sentences is known to recruit Broca’s area. Comparisons across brain imaging studies investigating similar hierarchical structures in different domains revealed that complex hierarchical structures that mimic those of natural languages mainly activate Broca’s area, that is, left Brodmann area (BA) 44/45, whereas hierarchically structured mathematical formulae, moreover, strongly recruit a more anteriorly located region BA 47.

In this perspective, Friedrich and Fiederichi (2009) provide brain-imaging evidence that the syntactic processing of abstract mathematical formulae, written in a first order language, is, indeed efficient and effective as a rule-based generation and decision process. However, it is remarkable, that the neural network involved, consisting of intra-parietal and prefrontal regions, only involves Broca’s area in a surprisingly selective way. This seems to imply that despite structural analogies of common and current formal languages, at the neural level, mathematics and natural language are processed differently, in principle.
Friederici et al.’s (2011) brain imaging results also indicate that the processing of hierarchically structured mathematical formulae and the processing complex syntactic hierarchies in language activates different areas of the brain. There would be a dichotomy to reason either geometrically or equivalently algebraically in mathematics. This would imply that Merge is sub-served crucially by two pathways, the one for syntax would pass via the ventrolateral prefrontal cortex, strongly relying on Broca’s area, and the other implementation for general reasoning which could integrate multi-sensory information via a dorsal frontoparietal network, with a strong involvement of the posterior parietal cortex and the angular gyrus.

Section summary

We argued that complex numerals as expressed in natural language and derived by recursive Merge. The recursion is indirect as enforced by Minimize Symmetrical Relations and the functional head relating the parts of these structures may not be pronounced, as enforced by Minimize Externalization. The hypothesis that mathematics emerged with Merge offers a rationale to the fact that Merge and recursion are observed in arithmetic as well as in language.
However, *Minimize Symmetrical Relations* and *Minimize Externalization* do not apply in the derivation of mathematical formulae. The fact that recursion is a property of the operations that derive both complex numerals and mathematical formulae, suggests the existence of a common biological basis for the two systems. The fact that recursion in complex numerals is restricted to indirect recursion as well as the fact that the mathematical operator relating the parts of complex numerals may not be pronounced in certain languages suggests that complex numerals would find their biological basis in the neuronal faculty that sub-serves grammar but would go beyond it.

5. CONCLUDING REMARKS

Hauser, Chomsky and Fitch (2002) argued that recursion sets human language apart from animal communication. It might be the case that bounded recursion is available in animal communication, while unbounded recursion is specific to the human language faculty. This divide may account for the fact that counting in animals is limited as well mathematical abilities.

Moreover, considering both the notion of recursion as the application of a rule to its own output and the notion of recursion as replication of the same, it might be the case that the human-animal discontinuity also lies in the ability to replicate a same categorical element unambiguously, via the intermediate merger of a functional head. Unbounded Indirect recursion would fall into language’ specificity. Theoretical support for this hypothesis comes from principles reducing complexity, such as *Minimize Symmetrical Relations*, viz., choice points, when they arise in the derivations, including with respect to the Select and Agree. We showed that nominal compounds in languages such as Brazilian Portuguese, as well as the derivation of complex numerals in different languages provide empirical support for this hypothesis. A functional head F must occupy the intermediate head position between the parts of nominal compounds and complex numerals otherwise their interpretation would not be legible at the CI interface. The fact that the intermediate functional F head may not be legible at the sensorimotor interface can be attributed, according to our hypothesis to the economy principle of *Minimize Externalization*. We leave for further research the question of why the functional head must be pronounced in some languages and not in others. A possible explanation would be that principles reducing complexity apply in the derivations if they do not override grammar internal principles, as we argued for in Di Sciullo and Aguero (2008) on independent grounds. Moreover, the fact that indirect recursion is not observed in mathematical computation suggests that indirect recursion is part of the domain specificity of human language.

We argued that principles of efficient computation apply in derivations of linguistic expressions, including in the derivation of complex numerals; whereas there is no evidence that such principles apply in the computation of mathematical formulae. There is no evidence for principles that would minimize symmetry in arithmetic computations. Thus, domain specificity of language can also be seen from the viewpoint of the principles of efficient computation. Results from neurosciences reveal that different, but not withstanding connected, areas of the brain compute language and mathematics.
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